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Feedback Control of Plants Driven by Nonlinear Actuators via Input-State Linearization

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A new systematic method to design a control law for a linear system with a nonlinear input is proposed. Conventionally, the inverse of the nonlinear function is often used to such problems, but it requires too much calculation to solve the nonlinear equation at each control step. Accordingly, we propose a new method that only applies the derivative of the nonlinear function. The essential idea of this method is to extend the original system with a pseudostate variable. This approach makes it possible to avoid to use the inverse of the nonlinear function and reduces the calculation load. Some numerical simulations are presented to illustrate the validity of this method.

I. Introduction

ALTHOUGH many controller design methods have been developed for linear systems, few methods address nonlinear systems. This paper focuses on systems that use nonlinear actuators. Systems with on-off actuators, such as reaction control system (RCS)-controlled spacecraft, or systems with actuators of limited output, such as spacecraft controlled by reaction wheels, are typical examples of systems of this type.

In general, the describing function method or the phase-plane analysis method is used to analyze controllers for single-input systems or low-order systems[1] with such types of actuators. In practice, a controller for a system of this type is designed for a continuous system, and then the control signal is pulse-width modulated.[2] However, these methods do not provide a systematic methodology for controller design.

Meanwhile, it is known in the field of optimal control that minimum time control problems yield bang-bang type controllers.[3] Because these results are obtained from the optimization problem, it is necessary to design a controller to follow the resultant optimum trajectory, but designing such a controller is generally difficult.

Consequently, it is desirable to develop a systematic methodology to design state-feedback controllers for nonlinear systems, similar to that for linear-continuous systems. This paper describes a control method that applies linearization[4] to the state-feedback control of nonlinear time-invariant systems.

We present a feedback control design method for an nth-order system with a nonlinear actuator in Sec. II, and some numerical results are shown in Sec. III, including an on-off controller example.

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II. Controller Design for a System Driven by a Nonlinear Actuator

A. Controller Design

A single-input nth-order system driven by a nonlinear actuator can be expressed as follows:

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + \boldsymbol{b}\psi(u) \tag{1}$$

where $\boldsymbol{x} \in R^n$, $u \in R$, and it is controllable. Suppose Eq. (1) is expressed in controllable canonical form as

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ & \vdots & & \ddots & \vdots \\ -\alpha_0 & -\alpha_1 & -\alpha_2 & \cdots & -\alpha_{n-1} \end{bmatrix}, \qquad \boldsymbol{b} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \tag{2}$$

In this equation, let $\psi(u)$ express the nonlinear characteristic of the actuator as a function of the controller output u. Assume $\psi(u)$ is differentiable by u any number of times and each of them hold nonzero values. Our objective is to find a state-feedback controller that stabilizes the system (1) and satisfies its control requirement.

One solution for this problem is the following method. If the nonlinear function ψ' is a nonzero function, then the system (1) can be linearized. Because Eqs. (1) and (2) show that the right hand of Eq. (1) can be decomposed into linear part $A\boldsymbol{x}$ and nonlinear part $\boldsymbol{b}\psi(u)$, it can be linearized.[5] Applying the method in Ref. 5 to this system, the variables and the input are converted as follows:

$$\begin{aligned} z_1 &= x_1 \\ &\vdots \\ z_n &= x_n \\ v &= -\alpha_0 x_1 - \alpha_1 x_2 \cdots - \alpha_{n-1} x_n + \psi(u) \end{aligned} \tag{3}$$

and we obtain the following linear system:

$$\begin{aligned} \dot{z}_1 &= z_2 \\ &\vdots \\ \dot{z}_n &= v \end{aligned} \tag{4}$$

In this case, the actual input u is calculated from the virtual input v.

$$u = \psi^{-1}(v + \alpha_0 x_1 + \alpha_1 x_2 + \cdots + \alpha_{n-1} x_n) \tag{5}$$

Here we propose a new method that does not apply the inverse function directly but uses dynamic nonlinear state feedback. The resulting controller in this framework stabilizes the closed-loop system, and the controller itself is stable. To use this dynamic nonlinear state-feedback method, we introduce the following extended system to realize the linearization:

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + \boldsymbol{b}\psi(x_{n+1}), \qquad \dot{x}_{n+1} = -\alpha x_{n+1} + \beta u^* \tag{6}$$

where x_{n+1} is a pseudostate variable and u^* is a pseudocontrol variable. Equation (6) can be written in the form:

$$\dot{\tilde{\boldsymbol{x}}} = \boldsymbol{f}(\tilde{\boldsymbol{x}}) + \boldsymbol{g}u^* \tag{7}$$

where

$$\tilde{\boldsymbol{x}} = \begin{bmatrix} \boldsymbol{x} \\ x_{n+1} \end{bmatrix} \tag{8}$$

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{bmatrix} A\boldsymbol{x} + \boldsymbol{b}\psi(x_{n+1}) \\ -\alpha x_{n+1} \end{bmatrix} = \begin{bmatrix} x_2 \\ \vdots \\ x_n \\ -\alpha_0 x_1 - \cdots - \alpha_{n-1} x_n + \psi(x_{n+1}) \\ -\alpha x_{n+1} \end{bmatrix}$$

$$\boldsymbol{g} = \begin{bmatrix} 0 \\ \beta \end{bmatrix} \tag{9}$$

In this case, if $\beta \neq 0$ and $\psi'(x_{n+1}) \neq 0$, then it can be linearized for all x_{n+1} (Appendix A).

A standard linearization method[4] is then applied to obtain the coordinate transformation $\boldsymbol{z} = \boldsymbol{z}(\boldsymbol{x})$ and the input transformation $u^* = p(\boldsymbol{x}) + q(\boldsymbol{x})v$. To apply this linearization method, let ∇_z denote the Jacobian matrix $\nabla_z = [\partial z_i / \partial x_j]$, that is,

$$\nabla_z = \begin{bmatrix} \nabla_{z_1} \\ \vdots \\ \nabla_{z_{n+1}} \end{bmatrix} \tag{10}$$

Lie brackets $[f, g] = \nabla_g f - \nabla_f g$ are defined as $ad_f^0 g := g$, $ad_f^i g := [f, ad_f^{i-1} g]$, and the Lie derivative with respect to $\boldsymbol{f}$ is defined by $L_f h = \nabla_h \boldsymbol{f}$, where h is a scalar function and $\boldsymbol{f}$ is a vector field.

As is described in Ref. 4, the coordinate transformation is obtained as

$$\begin{aligned} z_1 &= x_1 \\ &\vdots \\ z_{n+1} &= -\alpha_0 x_1 - \alpha_1 x_2 - \cdots - \alpha_{n-1} x_n + \psi(x_{n+1}) \end{aligned} \tag{11}$$

Because the input transformation is $u^* = (v - L_f^{n+1} z_1)/(L_g L_f^n z_1)$, we obtain

$$\begin{aligned} u^* = (1/\beta\psi')\big[&v - \alpha_0\alpha_{n-1}x_1 - (\alpha_1\alpha_{n-1} - \alpha_0)x_2 \cdots \\ &- \big(\alpha_{n-1}^2 - \alpha_{n-2}\big)x_n + \alpha_{n-1}\psi(x_{n+1}) + \alpha x_{n+1}\psi'\big] \end{aligned} \tag{12}$$

Then, substituting Eqs. (11) and (12) into Eqs. (7), (8), and (9) results in the linearized system given here:

$$\begin{aligned} \dot{z}_1 &= z_2 \\ &\vdots \\ \dot{z}_n &= z_{n+1} \\ \dot{z}_{n+1} &= v \end{aligned} \tag{13}$$

The system (13) is state controllable, and so we can design the controller

$$v = -Kz = -k_1 z_1 \cdots - k_{n+1} z_{n+1} \tag{14}$$

using linear control theories.

Although the control law designed in this manner uses pseudostate variables $z_1, z_2, \ldots, z_{n+1}$ in the linearized state space, it can be implemented as a simple state-feedback controller using the actual state variables $\boldsymbol{x}$ as described next.

The real input to the actuator with a nonlinear characteristic function $\psi(x_{n+1})$ is x_{n+1}, which can be obtained from Eq. (6). Using Eqs. (11), (14), and (12), the control variable u^* is given by

$$u^* = \frac{-\bar{k}_1 x_1 - \bar{k}_2 x_2 \cdots - \bar{k}_n x_n + (\alpha_{n-1} - k_{n+1})\psi + \alpha\psi' x_{n+1}}{\beta\psi'} \tag{15}$$

where

$$\bar{k}_i = \begin{cases} k_i - \alpha_{i-1}k_{n+1} + \alpha_{i-1}\alpha_{n-1} & i = 1 \\ k_i - \alpha_{i-1}k_{n+1} + \alpha_{i-1}\alpha_{n-1} - \alpha_{i-2} & i = 2, \ldots, n \end{cases} \tag{16}$$

are constants. Summarizing the second equation of Eqs. (6) and (15), the dynamic characteristic of the controller is

$$\dot{x}_{n+1} = -(\bar{k}_1 x_1 \cdots + \bar{k}_n x_n)/\psi' + (\alpha_{n-1} - k_{n+1})\psi/\psi' \tag{17}$$

Thus, the controller can solve Eq. (17) using $x_1, \ldots, x_n$ and outputs the signal x_{n+1} to the actuator.

B. Controller Properties

The closed loop resulting from this method is stable if there are no modeling errors in the control target. Moreover, if $k_{n+1} > \alpha_{n-1}$, it can be shown that the controller (17) itself is stable (Appendix B). This fact is important because it is undesirable to have an unstable transfer function in the loop even if the closed loop is stable.

When the system (1) was extended to Eq. (6), we added a first-order system with parameters α, β. However, the resultant controller (17) does not include these parameters. From this, we can see that the resultant controller is not dependent on the values of α, β; it simply requires $\beta \neq 0$. Furthermore, we obtain similar results even if we add a higher-order system.

Using the method of Ref. 5, it is possible to linearize the system only if the inverse function exists. However, the existence of the inverse function and the region where the inverse function exists depend entirely on the function ψ. In our proposed method, there is also a region where the variable transformation exists. For example, if $\alpha_0, \ldots, \alpha_{n-1} = 0$ and the output of $\psi(x_{n+1})$ is limited, the equations of coordinate transformation (11) show that $z_{n+1} = \psi(x_{n+1})$ has to be also limited. In this sense, there is no great difference between our method and that of Ref. 5. However, we can evaluate the existence of variable transformation from the behavior of the subsystem (17), which consists of an added variable and a pseudoinput. This gives us more flexibility to design a controller.

III. Example Applications

A. Example 1

We apply our method to a simple problem with a nonlinear function whose inverse cannot be obtained analytically. Consider a simple system expressed as

$$\dot{x}_1 = x_2, \qquad \dot{x}_2 = \tfrac{1}{3}u^3 + u^2 + 2u \tag{18}$$

In this case, ψ^{-1} cannot be described explicitly, and so it is difficult to apply the conventional method using ψ^{-1}. The conventional method requires solving the nonlinear equation to get ψ^{-1} at each control step. However, we can apply the proposed method.

First, this system is extended as

$$\dot{x}_1 = x_2, \qquad \dot{x}_2 = \psi(x_3), \qquad \dot{x}_3 = -x_3 + u^* \tag{19}$$

Applying the coordinate transformation

$$z_1 = x_1, \qquad z_2 = x_2, \qquad z_3 = \psi(x_3) \tag{20}$$

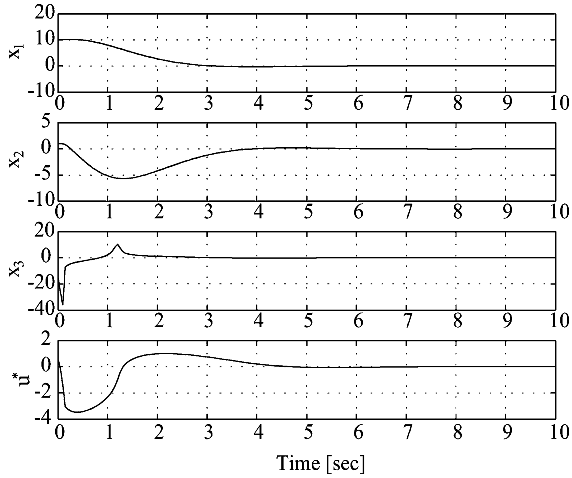

Fig. 1 Results of example 1.

the linearized system is

$$\dot{z}_1 = z_2, \qquad \dot{z}_2 = z_3, \qquad \dot{z}_3 = v \tag{21}$$

where

$$u^* = x_3 + v/\psi'(x_3) \tag{22}$$

From Eqs. (19), we can easily obtain the derivative of the nonlinear function as

$$\psi'(u) = u^2 + 2u + 2 \tag{23}$$

Note that ψ' obviously holds nonzero value.

Now, we can design a controller for this linearized system using appropriate control theory. In this example, the controller is designed as

$$v = -Kz \tag{24}$$

where $K = [4\ \ 6\ \ 4]$. Figure 1 shows the time evolution of this case. The initial states are $x_1(0) = 10.0$, $x_2(0) = 1.0$.

This example shows that the proposed method works effectively and has an advantage of applicability comparing to the conventional method.

B. Example 2

The next example is also a simple problem with a nonlinear function the derivative of which is zero at some points. Consider a simple system expressed as

$$\dot{x}_1 = x_2, \qquad \dot{x}_2 = u^3 - u = \psi(u) \tag{25}$$

In this case, ψ^{-1} cannot be described explicitly, again. However, we can apply the proposed method.

First, this system is extended as

$$\dot{x}_1 = x_2, \qquad \dot{x}_2 = \psi(x_3), \qquad \dot{x}_3 = -x_3 + u^* \tag{26}$$

Note that ψ' must be nonzero. Then, ψ is approximated as

$$\tilde{\psi} = \begin{cases} \alpha(u - u_1) + \psi_1 & u_1 \le u \le u_2 \\ \alpha(u - u_3) + \psi_3 & u_3 \le u \le u_4 \\ u^3 - u & \text{otherwise} \end{cases} \tag{27}$$

where α is a small positive number and

$$u_1 = \tfrac{1}{2}\sqrt{(1-\alpha)/3} - \sqrt{(5\alpha + 3)}\Big/2 \tag{28}$$

$$u_2 = -\sqrt{(1-\alpha)/3} \tag{29}$$

$$u_3 = \sqrt{(1-\alpha)/3} \tag{30}$$

$$u_4 = -\tfrac{1}{2}\sqrt{(1-\alpha)/3} + \sqrt{(5\alpha + 3)}\Big/2 \tag{31}$$

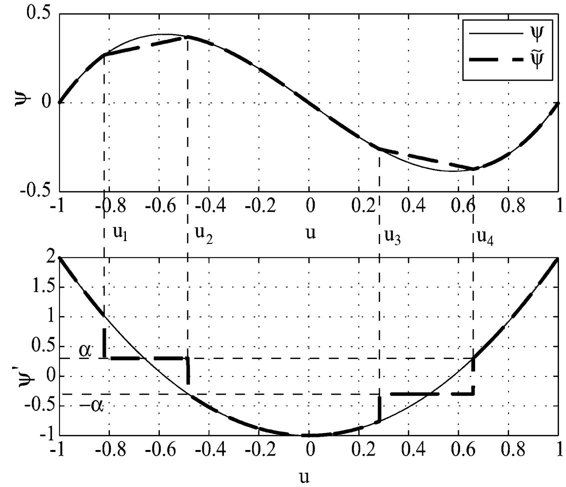

Fig. 2 ψ and its approximation $\tilde{\psi}$.

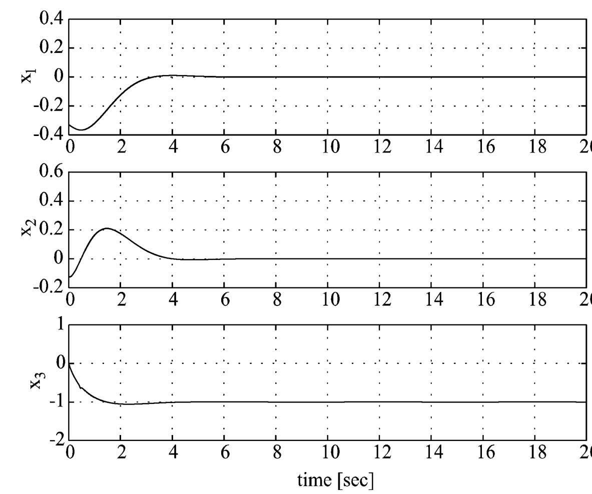

Fig. 3 Results of example 2.

$$\psi_1 = u_1^3 - u_1 \tag{32}$$

$$\psi_3 = u_3^3 - u_3 \tag{33}$$

The constants of u_1 to u_4 are determined so as to keep $|\psi'(u)| \ge \alpha$. Equation (17) indicates that the size of control input $u = x_{n+1}$ is governed by the size of $1/\psi'$. Therefore, the size of α is determined with consideration of the size of u. Figure 2 shows the original nonlinear function ψ (solid) and its approximation $\tilde{\psi}$ (dashed). In this figure, $\alpha = 0.3$. This is intentionally set to illustrate the difference between ψ and $\tilde{\psi}$ clearly.

Then, we obtain the derivative of $\tilde{\psi}$ as

$$\tilde{\psi}' = \begin{cases} \alpha & u_1 \le u \le u_2 \\ \alpha & u_3 \le u \le u_4 \\ 3u^2 - 1 & \text{otherwise} \end{cases} \tag{34}$$

Applying the coordinate transformation

$$z_1 = x_1, \qquad z_2 = x_2, \qquad z_3 = \psi(x_3) \tag{35}$$

the linearized system is

$$\dot{z}_1 = z_2, \qquad \dot{z}_2 = z_3, \qquad \dot{z}_3 = v \tag{36}$$

Then we can design a controller as is described in example 1. The same controller is used, and the initial states are $x_1(0) = -0.127$, $x_2(0) = -0.1748$, $\alpha = 0.05$, and time step for the controller is 0.05 s.

Figure 3 shows the results of this case. This shows that the proposed method is applicable with approximation of the nonlinear function.

C. Example 3

Our method is applied to the attitude control of a rigid satellite with RCS. The equations of motion around one axis are generally expressed as

$$\dot{x}_1 = x_2, \qquad \dot{x}_2 = F\,\mathrm{sign}(u) \tag{37}$$

where x_1 is the angle, x_2 is the rotational velocity, u is the RCS command, F is RCS torque, and the moment of inertia is unity. We construct an extended system as follows:

$$\dot{x}_1 = x_2, \qquad \dot{x}_2 = F\psi(x_3), \qquad \dot{x}_3 = a(u^* - x_3) \tag{38}$$

For this extended system, the nonlinear function is approximated by $\psi \approx \psi_\rho(x_3) = (2/\pi)\tan^{-1}(\rho x_3)$, and the following coordinate transformation is applied:

$$z_1 = x_1, \qquad z_2 = x_2, \qquad z_3 = F\psi_\rho(x_3) \tag{39}$$

We can now linearize this system with the input transformation $u^* = v/[a\psi'_\rho(x_3)] + x_3$, where $\psi'_\rho(x_3) = (2/\pi)\rho/(1+\rho^2 x_3^2)$. The linearized system is

$$\dot{z}_1 = z_2, \qquad \dot{z}_2 = z_3, \qquad \dot{z}_3 = v \tag{40}$$

Linear control theory is applied to this linearized system, and the resulting state-feedback controller is

$$v = -k_1 z_1 - k_2 z_2 - k_3 z_3 \tag{41}$$

In this example, the initial states are $x_1(0) = 2$, $x_2(0) = 0$ and parameters are $a = 2/\pi$, $F = 1$, $\rho = 1000$. The control gains are $k_1 = k_2 = 5a^2$, $k_3 = 5a$.

The simulation result is shown in Fig. 4. From the top, the plots show the time histories of angle, angular velocity, RCS command, and RCS torque. From this figure, it follows that the system is stabilized by the bang-bang type torque of the actuators. The steady-state response of the system is a limit cycle by on-off control.

In addition, we can obtain a new controller with a dead-band, which suppresses chattering in the next example.

D. Example 4

In this example, an appropriate dead-band is added to the preceding problem to suppress chattering caused by the RCS command. An ideal dead-band signum function is defined as

$$\psi(u) = \begin{cases} -1 & u < -\delta \\ 0 & |u| \le \delta \\ 1 & u > \delta \end{cases} \tag{42}$$

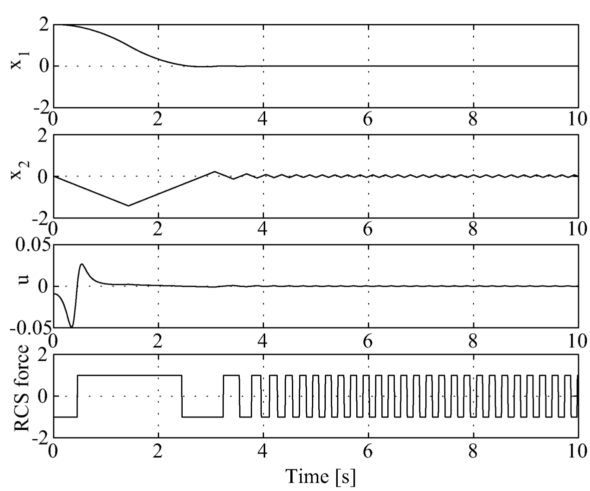

Fig. 4 Results of example 3.

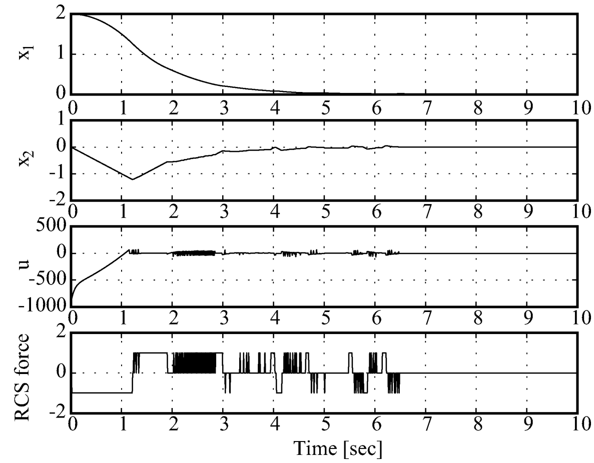

Fig. 5 Results of example 4.

where 2δ is the width of the dead-band. However, the derivative of this function in the dead-band $u \in [-\delta, \delta]$ is $\psi'(u) = 0$, and Eq. (17) cannot be calculated because the inverse of the derivative does not exist. Therefore, Eq. (42) is modified so that it has a slight slope ($\alpha > 0$) in the dead-band, and the slope of $\tan^{-1}$ is replaced with an appropriate constant (α). By these manipulations, the signum function is changed to the form of

$$\psi_\rho(u) = \begin{cases} \alpha(u+u_1) - (2/\pi)\tan^{-1}(\rho u_1) & u < -\delta - u_1 \\ (2/\pi)\tan^{-1}[\rho(u+\delta)] - \alpha\delta & -\delta - u_1 \le u < -\delta \\ \alpha u & |u| \le \delta \\ (2/\pi)\tan^{-1}[\rho(u-\delta)] + \alpha\delta & \delta \le u < \delta + u_1 \\ \alpha(u-u_1) + (2/\pi)\tan^{-1}(\rho u_1) & u \ge \delta + u_1 \end{cases} \tag{43}$$

where $u_1 = (1/\rho)\sqrt{[2\rho/(\pi\alpha) - 1]}$, which is determined to connect ψ_ρ and ψ'_ρ at $u = \pm(\delta + u_1)$. The derivative of this function is

$$\psi'_\rho(u) = \begin{cases} \dfrac{(2/\pi)\rho}{(1+\rho^2(|u|-\delta)^2)} & \delta \le |u| < \delta + u_1 \\ \alpha & \text{otherwise} \end{cases} \tag{44}$$

Figure 5 shows the result of a simulation in which ψ_ρ in Eq. (39) is replaced with ψ_ρ in Eq. (43). This figure also shows the time histories of angle, angular velocity, RCS command, and RCS torque from the top. This result shows that the dead-band works effectively and the system is stabilized without limit cycle of RCS command. In this example, control gains are designed as the optimum regulator, and their values are $k_1 = 100$, $k_2 = 114.1$, and $k_3 = 15.1$.

IV. Conclusions

This paper presents an input-state linearization method to allow design of state-feedback controllers for single-input linear systems. This method does not apply the inverse of the nonlinear function of the systems, which often requires too much computational load to solve nonlinear equations, and this is the effective point of the proposed method comparing to the conventional method.

Appendix A: Possibility of Linearization

To prove that the nonlinear system (7) can be linearized, it is sufficient to show that the vector field $\{g, ad_f g, \ldots, ad_f^n g\}$ is linearly independent (controllability) and that a set of $\{g, ad_f g, \ldots, ad_f^{n-1} g\}$ is involutive. The controllability is directly shown by the calculation of the rank of the matrix. Because the first $(n-i-1)$ components of $ad_f^i g$ are 0 and the $(n-i)$th component of that is $(-1)^{i-1}\beta\psi'$, the controllability matrix is

$$\begin{bmatrix} g & ad_f g & \cdots & ad_f^n g \end{bmatrix} = \begin{bmatrix} 0 & & & (-1)^{n-1}\beta\psi' \\ & & \cdots & \\ & (-1)^{i-1}\beta\psi' & & \\ \cdots & & & \\ \beta & & & * \end{bmatrix} \tag{A1}$$

From this equation, if $\beta \neq 0$ and $\psi' \neq 0$,

$$\mathrm{rank}\begin{bmatrix} g & ad_f g & \cdots & ad_f^n g \end{bmatrix} = n + 1 \tag{A2}$$

for all x_{n+1}. The controllability is proved.

To show that the set of $\{g, ad_f g, \ldots, ad_f^{n-1} g\}$ is involutive, let this set be $\{\boldsymbol{h}_1, \ldots, \boldsymbol{h}_n\}$. Then, Lie brackets of $\boldsymbol{h}_i$ and $\boldsymbol{h}_j$ become

$$[h_i, h_j] = \nabla_{f_j} f_i - \nabla_{f_i} f_j \tag{A3}$$

Note that all elements on the right-hand side have the property that all elements in the first row are 0. This yields

$$\mathrm{rank}\begin{bmatrix} g & ad_f g & \cdots & ad_f^{n-1} g \end{bmatrix} = \mathrm{rank}\begin{bmatrix} g & ad_f g & \cdots & ad_f^{n-1} g & [h_i, h_j] \end{bmatrix} = n \tag{A4}$$

Consequently, if $\beta \neq 0$ and $\psi' \neq 0$, the set is involutive. □

Appendix B: Stability of the Controller

To show the stability of the controller, consider the expression in Eq. (17),

$$\dot{x}_{n+1} = -(\bar{k}_1 x_1 \cdots + \bar{k}_n x_n)/\psi' + (\alpha_{n-1} - k_{n+1})\psi/\psi' \tag{B1}$$

with input terms $x_1, \ldots, x_n$. Rewriting this equation without the input terms yields

$$\dot{x}_{n+1} = (\alpha_{n-1} - k_{n+1})\psi/\psi' \tag{B2}$$

One candidate for the Lyapunov function is

$$V = \psi^2 > 0 \qquad \text{for} \qquad x_{n+1} \neq 0 \tag{B3}$$

Using this function, we can see that

$$\begin{aligned} \dot{V} &= 2\psi(x_{n+1})\dot{\psi}(x_{n+1}) \\ &= 2\psi(x_{n+1})\psi'(x_{n+1})\frac{\partial x_{n+1}}{\partial t} \\ &= -2(k_{n+1} - \alpha_{n-1})\psi^2(x_{n+1}) \end{aligned} \tag{B4}$$

Thus, the controller is stable if $k_{n+1} > \alpha_{n-1}$. □

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